CSC/MTH 228 Discrete Mathematical Structures

Exam II

1. (2 points) **Binary, Decimal, and Hexadecimal Conversion**
   
   (a) Convert **ACE** to binary and decimal
   
   (b) Convert **417** to binary and hexadecimal

2. (2 points) **Hexadecimal Addition and Multiplication**
   
   Find the sum and product of these two numbers: **1AE** and **BBC**
   
   Express your answers as a hexadecimal number.

3. (2 points) **Modular Exponentiation**
   
   \[ 3^{21} \pmod{29} \]

4. (2 points) **Find GCD and LCM**
   
   Find the greatest common divisor and the least common multiple of these two integers: **76** and **36**.

5. (2 points) **Solve Linear Congruence**
   
   \[ 55x \equiv 34 \pmod{89} \]

6. (2 points) **Mathematical Induction**
   
   Prove that (using math induction):
   
   \[ \sum_{i=1}^{n} i \cdot i! = (n + 1)! - 1 \]
7. (2 points) **Counting**

Each DNA sequence contains sub-elements, each of which is adenine (A), cytosine (C), guanine (G), or thymine (T).

How many 5-element DNA sequences

(a) end with A?
(b) start with T and end with G?
(c) contain only A and T?
(d) do not contain C?

8. (2 points) **Counting II**

A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages?

9. (2 points) **Pigeonhole Principle**

No one has more than 1 million hairs on the head. And the population of New York City was 8,804,190 in 2020. Show there had to be at least how many people in New York City in 2020 with the same number of hairs on their heads?

10. (2 points) **Permutation and Combination**

Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

11. (2 points) **Bonus Question**

You have 1000 bottles of wine for a birthday party. 20 hours before the party, the winery indicates 1 bottle of wine is filled with poison, without telling you which bottle. You have 10 lab mice to test this on. The poison is so strong that it’ll take only 18 hours to kill the mice who drink it. Is there a way that you can find the poisoned bottle using the 10 mice before the party?