
Syntax

t ::=

true

false

if t then t else t

terms:

constant true

constant false

conditional

Syntax

t ::=

true

false

if t then t else t

v ::=

true

false

terms:

constant true

constant false

conditional

values:

true value

false value

Evaluation

$t \rightarrow t'$

if true then t₂ else t₃ → t₂ (E-IFTRUE)

if false then t₂ else t₃ → t₃ (E-IFFALSE)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3}$$
 (E-IF)

Syntax

$t ::=$

true

false

if t then t else t

terms:
constant true
constant false
conditional

$v ::=$

true

false

values:
true value
false value

Evaluation

$t \rightarrow t'$

if true then t_2 else $t_3 \rightarrow t_2$ (E-IFTRUE)

if false then t_2 else $t_3 \rightarrow t_3$ (E-IFFALSE)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-IF})$$

New syntactic forms

$t ::=$...

0

succ t

pred t

iszero t

terms:
constant zero
successor
predecessor
zero test

$v ::=$...

nv

values:
numeric value

$nv ::=$

0

succ nv

numeric values:
zero value
successor value

New evaluation rules

$t \rightarrow t'$

$$\frac{t_1 \rightarrow t'_1}{\text{succ } t_1 \rightarrow \text{succ } t'_1} \quad (\text{E-SUCC})$$

pred 0 $\rightarrow$ 0 (E-PREDZERO)

pred (succ nv_1) $\rightarrow$ nv_1 (E-PREDSUCC)

$$\frac{t_1 \rightarrow t'_1}{\text{pred } t_1 \rightarrow \text{pred } t'_1} \quad (\text{E-PRED})$$

iszero 0 $\rightarrow$ true (E-ISZEROZERO)

iszero (succ nv_1) $\rightarrow$ false (E-ISZEROSUCC)

$$\frac{t_1 \rightarrow t'_1}{\text{iszero } t_1 \rightarrow \text{iszero } t'_1} \quad (\text{E-ISZERO})$$

badnat ::=
wrong
true
false

badbool ::=
wrong
nv

non-numeric normal forms:

run-time error

constant true

constant false

non-boolean normal forms:

run-time error

numeric value

and we augment the evaluation relation with the following rules:

if badbool then t_1 else $t_2 \rightarrow \text{wrong}$ (E-IF-WRONG)

succ badnat $\rightarrow \text{wrong}$ (E-SUCC-WRONG)

pred badnat $\rightarrow \text{wrong}$ (E-PRED-WRONG)

iszero badnat $\rightarrow \text{wrong}$ (E-ISZERO-WRONG)