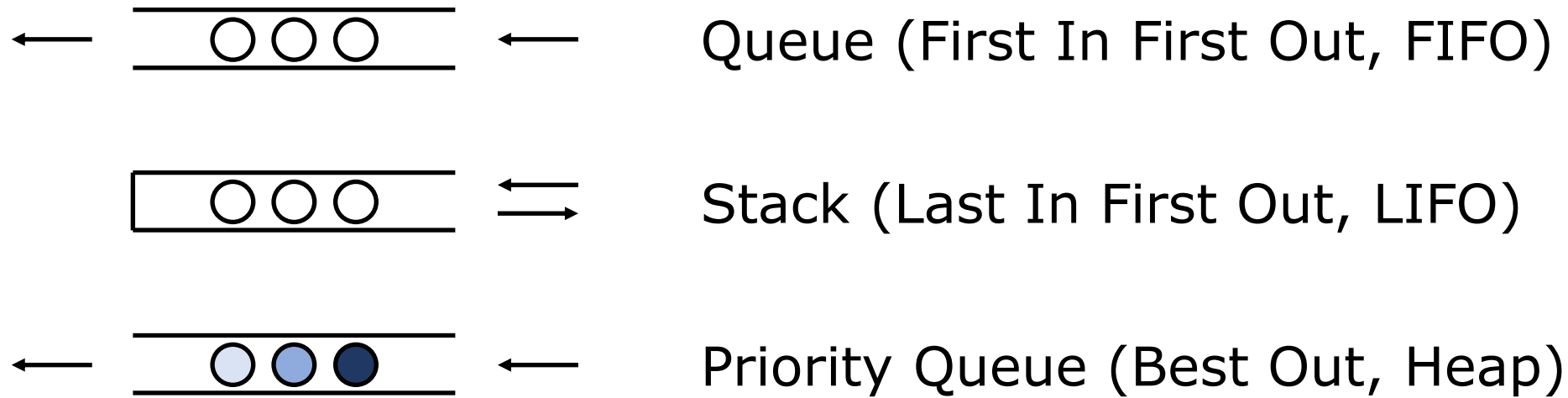


# Search Algorithms

Cong Chen

# 0. Reviewing data structures



key	value
314	1.5
926	5.3
...	...

Dictionary (Hash Table)

# Configurational Search Strategies

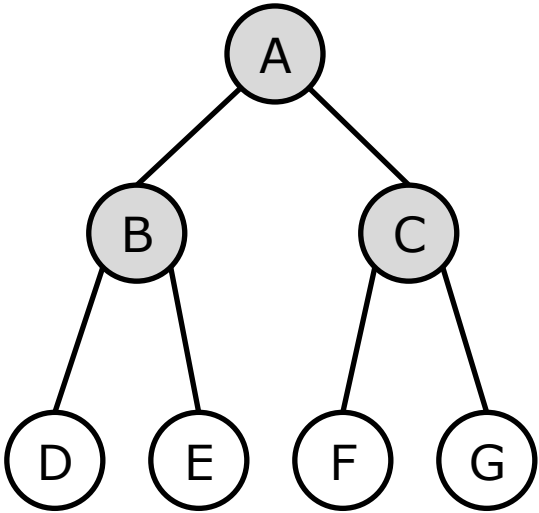
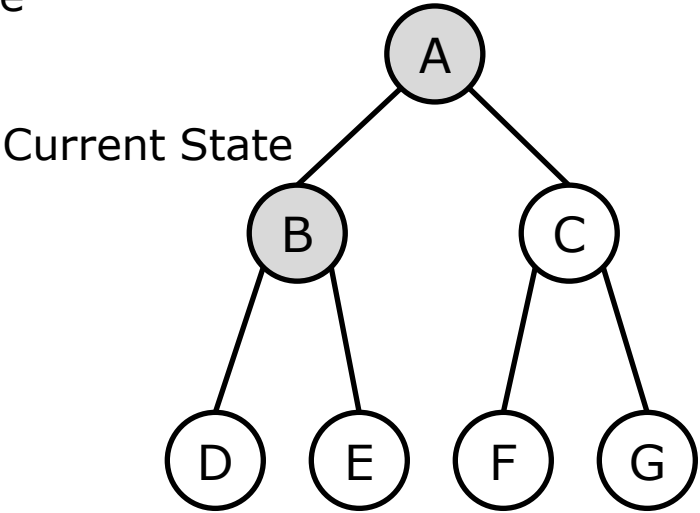
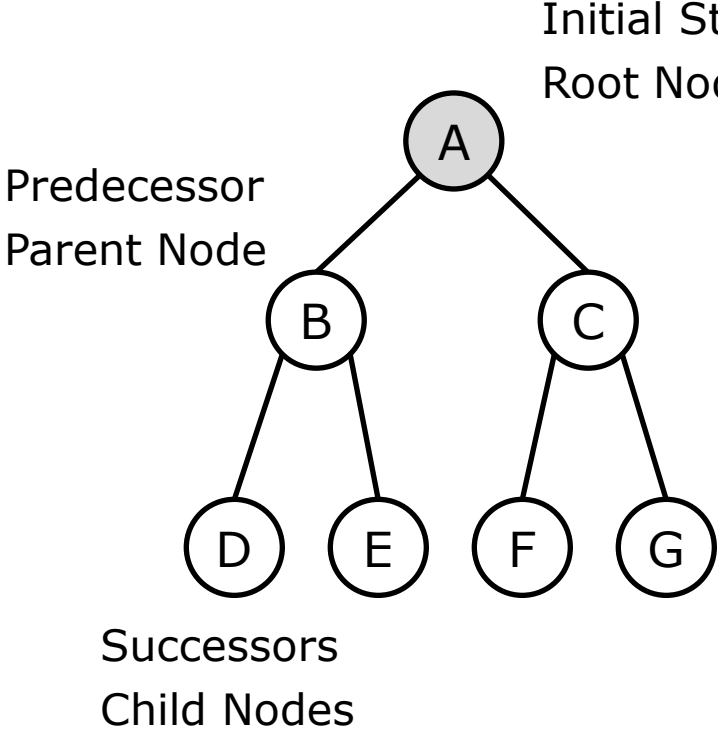


In a restaurant: Starter? Entrée?  
Medium-Rare? Sides? Drinks? Dessert?

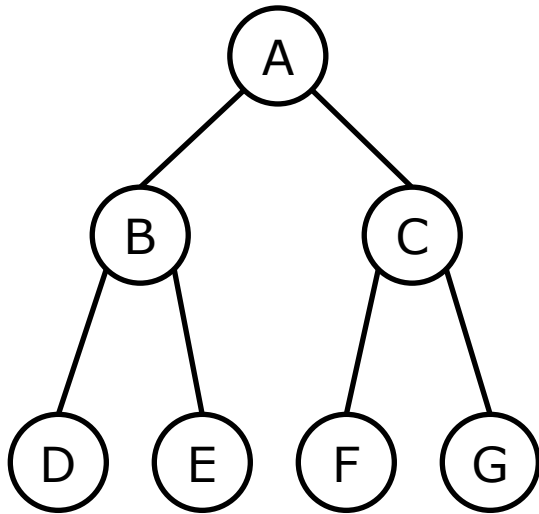
Deciding a search strategy: Depth or  
Best? One solution or M solutions?  
Heuristics? Tree? And-or?

# 1. Traversal

Search algorithms all share this basic structure; they vary primarily according to how they choose which state to expand next—the so-called **search strategy**.



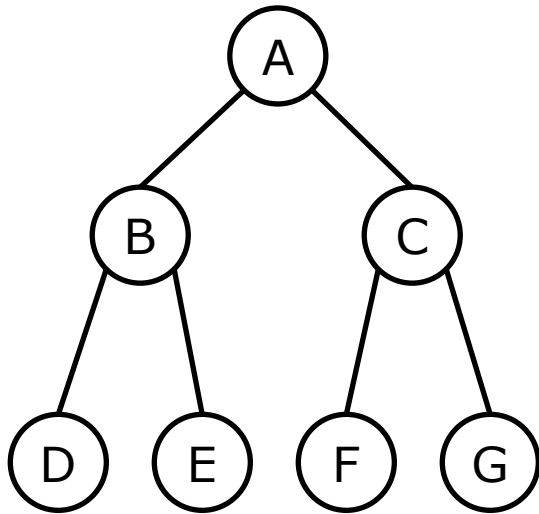
# Breadth First Traversal



Step	Queue
0	
1	<b>A</b>
2	<b>B C</b>
3	<b>C D E</b>
4	<b>D E F G</b>
5	<b>E F G</b>
6	<b>F G</b>
7	<b>G</b>
8	

```
Q.push(init);  
  
while ( not Q.empty() )  
{  
    cur = Q.pop();  
    Q.push( cur.succs() );  
}
```

# Depth First Traversal

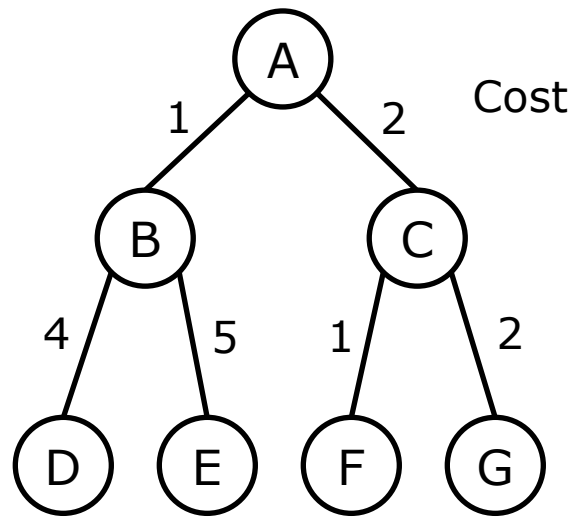


Step	Stack
0	
1	<b>A</b>
2	<b>B C</b>
3	<b>B F G</b>
4	<b>B F</b>
5	<b>B</b>
6	<b>D E</b>
7	<b>D</b>
8	

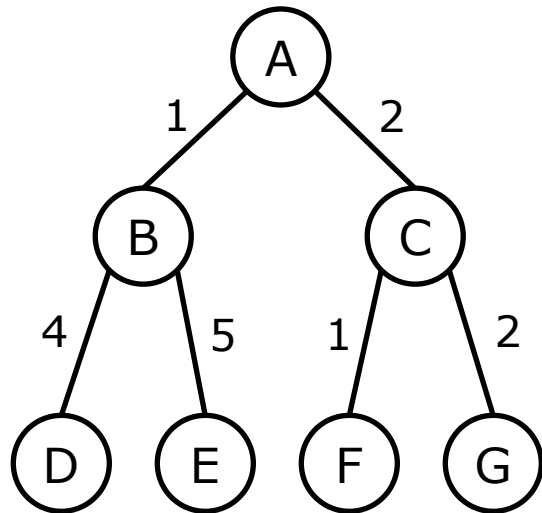
Memory-efficient

```
T.push(init);  
  
while ( not T.empty() )  
{  
    cur = T.pop();  
    T.push( cur.succs() );  
}
```

# Informed Traversal



# Best First Traversal



Step	Priority Q
0	
1	<b>A<sup>0</sup></b>
2	<b>B<sup>1</sup> C<sup>2</sup></b>
3	<b>C<sup>2</sup> D<sup>5</sup> E<sup>6</sup></b>
4	<b>F<sup>3</sup> G<sup>4</sup> D<sup>5</sup> E<sup>6</sup></b>
5	<b>G<sup>4</sup> D<sup>5</sup> E<sup>6</sup></b>
6	<b>D<sup>5</sup> E<sup>6</sup></b>
7	<b>E<sup>6</sup></b>
8	

```
PQ.push(init);
```

```
while ( not PQ.empty() )
```

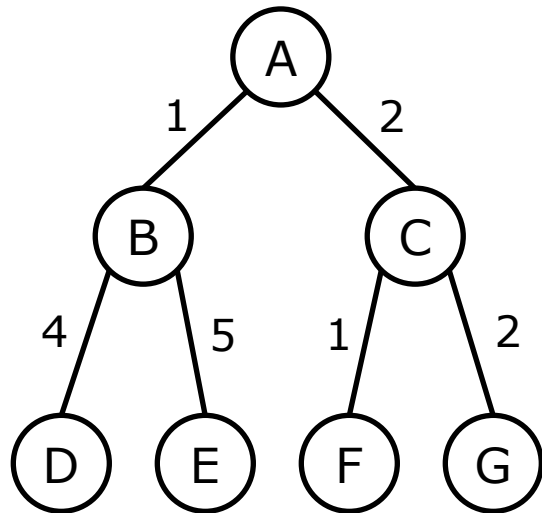
```
{
```

```
    cur = PQ.pop();
```

```
    PQ.push( cur.succs() );
```

```
}
```

# Depth First Traversal +



Step	Stack
0	
1	<b>A<sup>0</sup></b>
2	C <sup>2</sup> <b>B<sup>1</sup></b>
3	C <sup>2</sup> E <sup>6</sup> <b>D<sup>5</sup></b>
4	C <sup>2</sup> <b>E<sup>6</sup></b>
5	<b>C<sup>2</sup></b>
6	G <sup>4</sup> <b>F<sup>3</sup></b>
7	<b>G<sup>4</sup></b>
8	

```
T.push(init);
```

```
while ( not T.empty() )
```

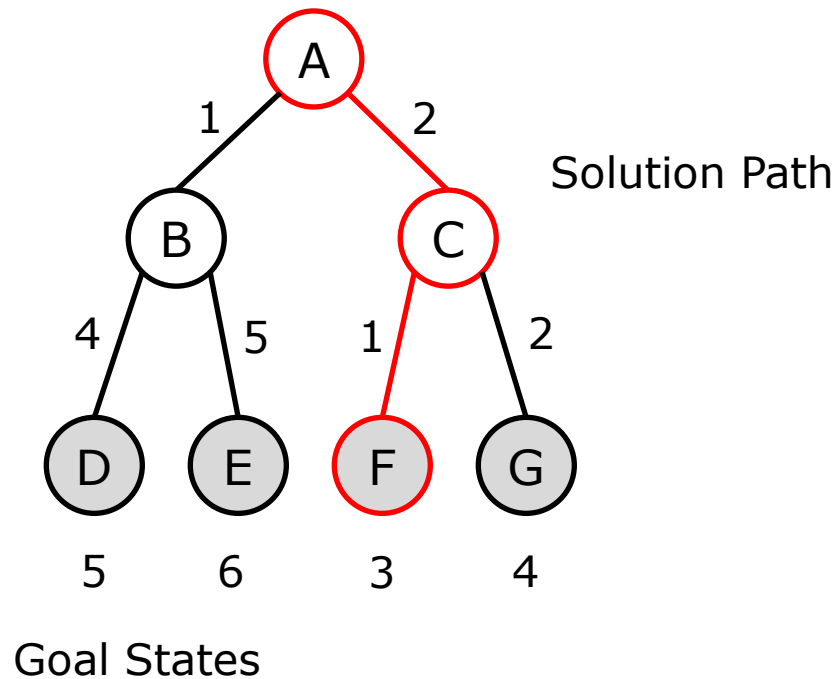
```
{
```

```
    cur = T.pop();
```

```
    T.push(cur.succs().sort(R));
```

```
}
```

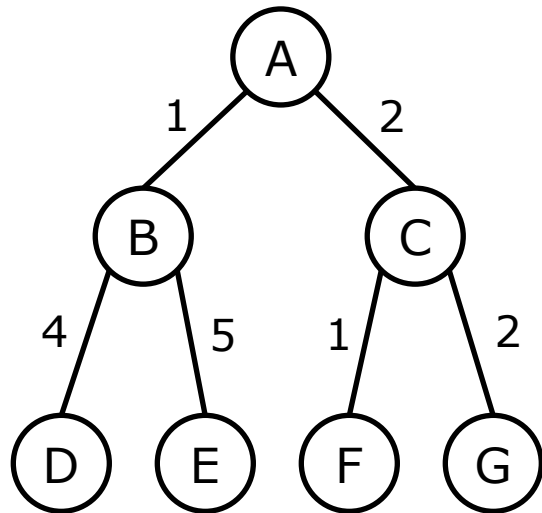
## 2. Search Tasks: 1-Best Solution VS M-Best Solutions



1-Best Solution: Find the best one from the goal state set

M-Best Solutions: Find top M best from the goal state set

# Best First Search – 1-Best



Step	Priority Q
0	
1	<b>A<sup>0</sup></b>
2	<b>B<sup>1</sup> C<sup>2</sup></b>
3	<b>C<sup>2</sup> D<sup>5</sup> E<sup>6</sup></b>
4	<b>F<sup>3</sup> G<sup>4</sup> D<sup>5</sup> E<sup>6</sup></b>
<del>5</del>	<del><b>G<sup>4</sup> D<sup>5</sup> E<sup>6</sup></b></del>
<del>6</del>	<del><b>D<sup>5</sup> E<sup>6</sup></b></del>
<del>7</del>	<del><b>E<sup>6</sup></b></del>
<del>8</del>	<del></del>

```
PQ.push(init);
```

```
while ( not PQ.empty() )
{
```

```
    cur = PQ.pop();
```

```
    if ( cur.goal() )
```

```
    {
```

```
        output(cur);
```

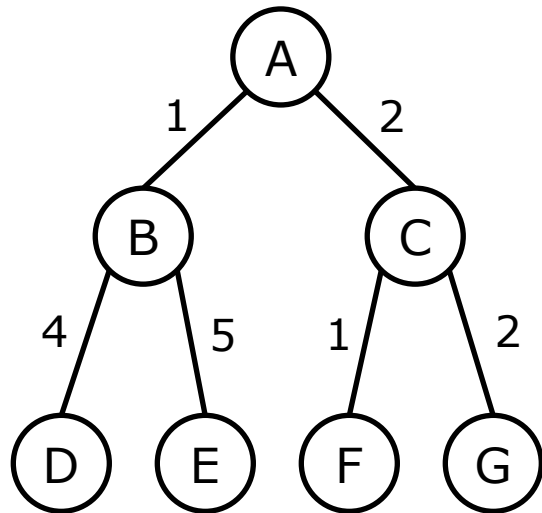
```
        break;
```

```
    }
```

```
    PQ.push( cur.succs() );
```

```
}
```

# Depth First Traversal – 1-Best



Step	Stack
0	
1	<b>A</b> <sup>0</sup>
2	C <sup>2</sup> <b>B</b> <sup>1</sup>
3	C <sup>2</sup> E <sup>6</sup> <b>D</b> <sup>5</sup>
4	C <sup>2</sup> <b>E</b> <sup>6</sup>
5	<b>C</b> <sup>2</sup>
6	G <sup>4</sup> <b>F</b> <sup>3</sup>
7	<b>G</b> <sup>4</sup>
8	

```

T.push(init);
best = ∞;

while ( not T.empty() )
{
    cur = T.pop();

    if ( cur.goal() )
    {
        if ( cur < best ) best = cur;
    }

    succs = cur.succs();
    succs.sort(Reverse);
    T.push(succs);
}
output(best)
  
```

# on\_goal()

```
PQ.push(init);  
  
while ( not PQ.empty() )  
{  
    cur = PQ.pop();  
  
    if ( cur.goal() )  
        signal = on_goal(cur);  
    ...  
  
    PQ.push( cur.succs() );  
}
```

Goal State Stream:  
(Best First)

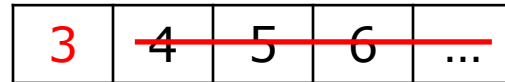
3	4	5	6	...
---	---	---	---	-----

Goal State Stream:  
(Depth First)

5	6	3	4	...
---	---	---	---	-----

# on\_goal() – 1-Best

Goal State Stream:  
(Best First)



Output the first solution and stop

Goal State Stream:  
(Depth First)



Updating a current best solution



# on\_goal() – M-Best

Goal State Stream:  
(Best First)

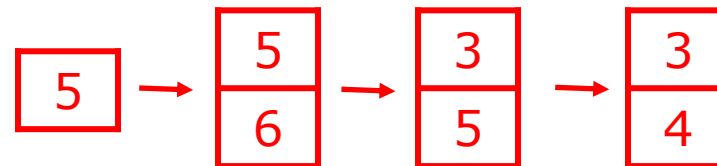


Output the first M=2 solutions and stop

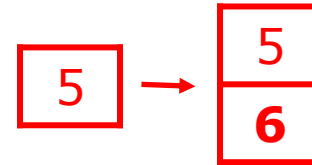
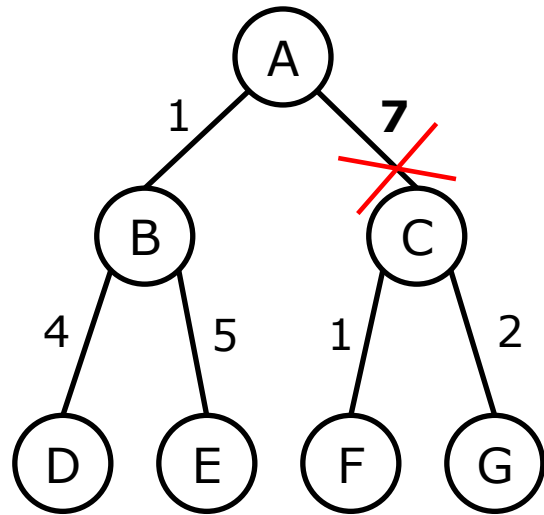
Goal State Stream:  
(Depth First)



Updating a current best solution list  
(max size = 2)

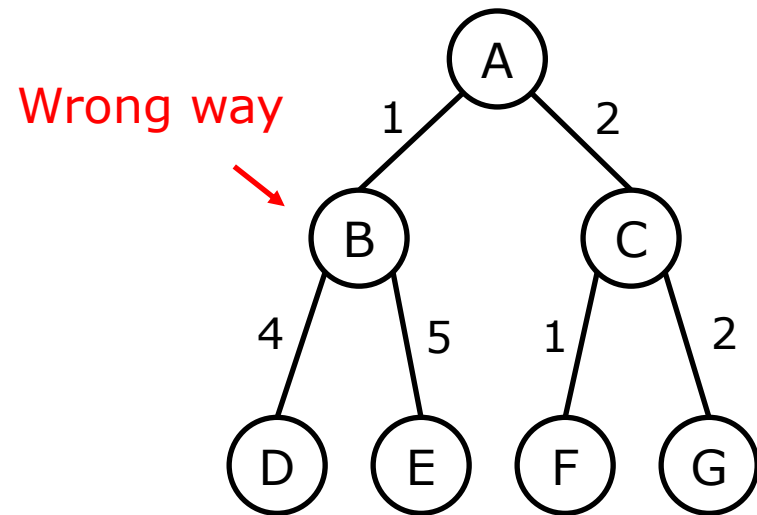


# Depth First Pruning



Start pruning when we have already  
got  $M=2$  solutions

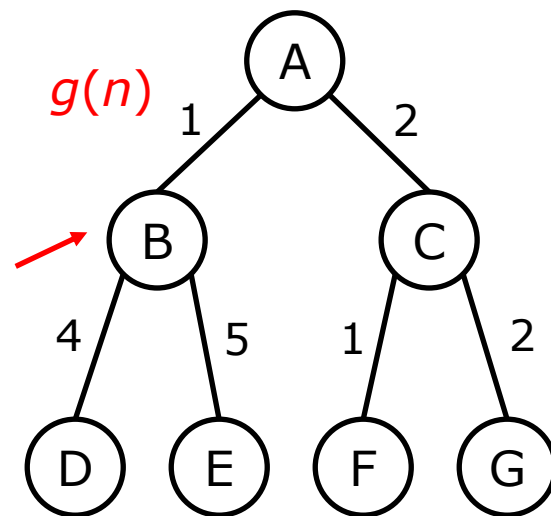
### 3. Heuristic Functions



Step	Priority Q
0	
1	<b>A<sup>0</sup></b>
2	<b>B<sup>1</sup> C<sup>2</sup></b>
3	<b>C<sup>2</sup> D<sup>5</sup> E<sup>6</sup></b>
4	<b>F<sup>3</sup> G<sup>4</sup> D<sup>5</sup> E<sup>6</sup></b>
5	<b>G<sup>4</sup> D<sup>5</sup> E<sup>6</sup></b>
6	<b>D<sup>5</sup> E<sup>6</sup></b>
7	<b>E<sup>6</sup></b>
8	

# Heuristic Functions (Addition)

Estimate at least 4  
in the future:  $h(n)$

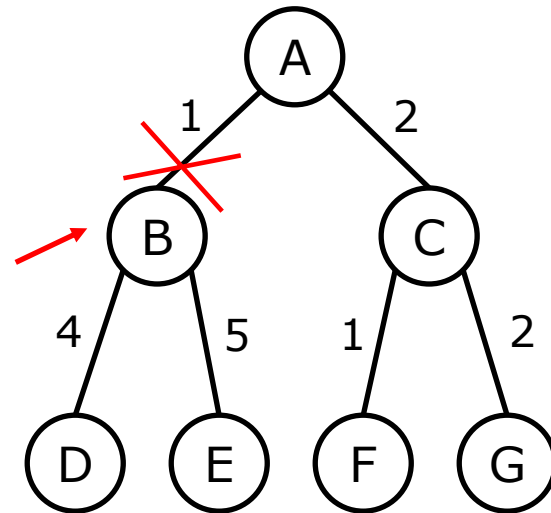


Step	Priority Q
0	
1	<b>A<sup>0</sup></b>
2	<b>C<sup>2+0</sup> B<sup>1+4</sup></b>
3	...

$$f(n) = g(n) + h(n)$$

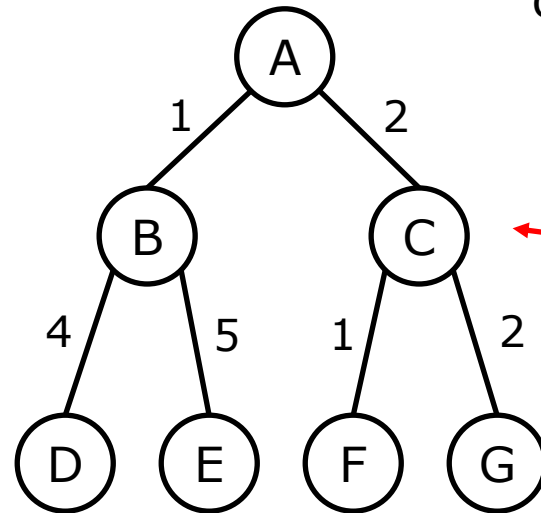
# Heuristics for pruning

Estimate at least 4  
in the future:  $h(n)$



# Admissible Heuristics

An **admissible** heuristic never overestimates the cost to reach the goal

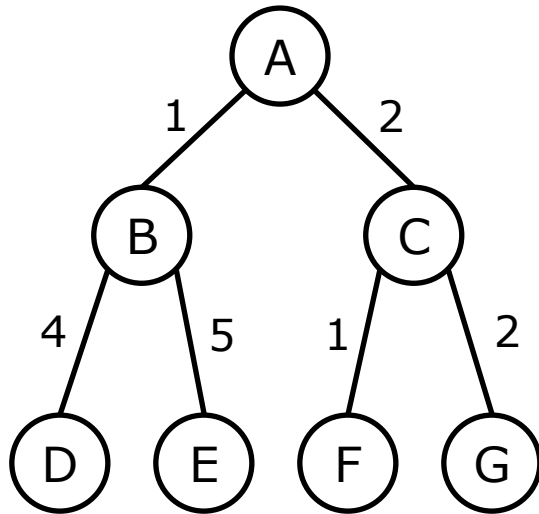


Can we say "At least 4"?

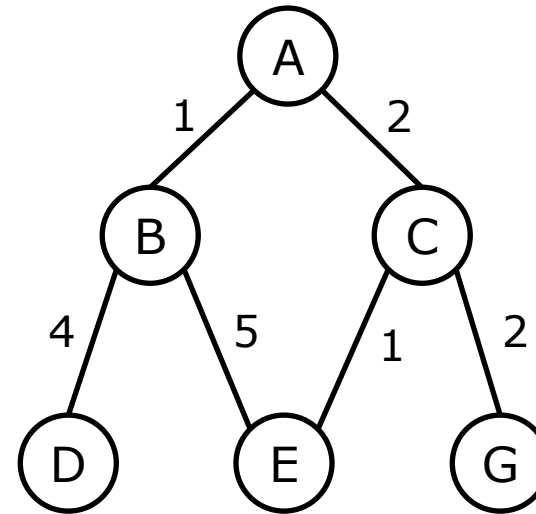
# 4. Graph Search Space

*"Those that fail to learn from history  
are doomed to repeat it."*

-- Winston Churchill



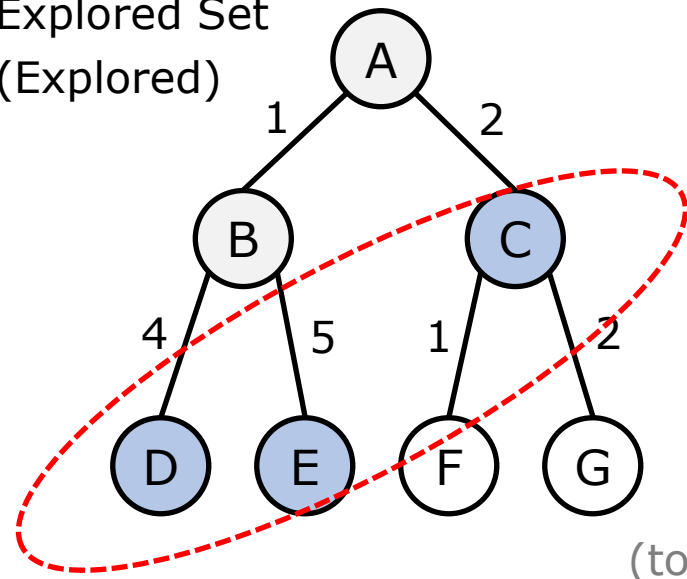
Tree  
No loops



Graph  
have loops

# Open List and Closed List

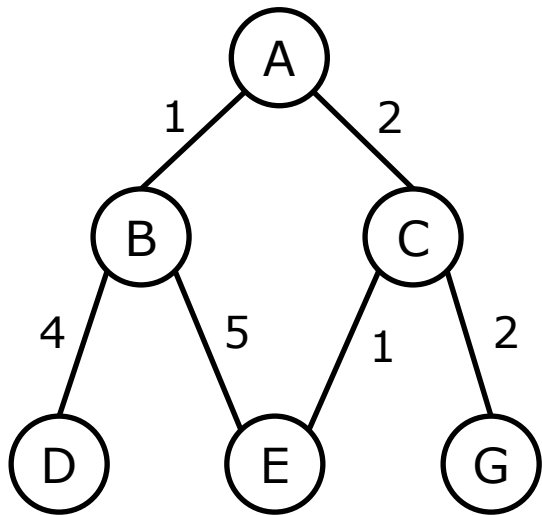
Closed List  
Explored Set  
(Explored)



Open List  
Frontier  
(Exploring)

Step	Priority Q
0	
1	<b>A<sup>0</sup></b>
2	<b>B<sup>1</sup> C<sup>2</sup></b>
3	<b>C<sup>2</sup> D<sup>5</sup> E<sup>6</sup></b>
4	<b>F<sup>3</sup> G<sup>4</sup> D<sup>5</sup> E<sup>6</sup></b>
5	<b>G<sup>4</sup> D<sup>5</sup> E<sup>6</sup></b>
6	<b>D<sup>5</sup> E<sup>6</sup></b>
7	<b>E<sup>6</sup></b>
8	

# Open List and Closed List

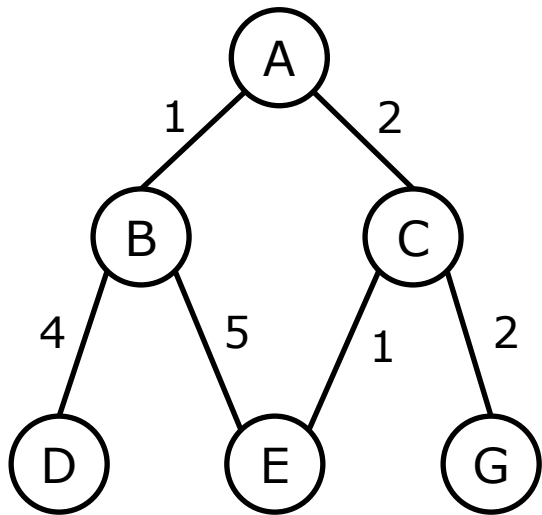


Step	Open
0	
1	<b>A<sup>0</sup></b>
2	<b>B<sup>1</sup>C<sup>2</sup></b>
3	<b>D<sup>5</sup>E<sup>6</sup>C<sup>2</sup></b>
4	<b>D<sup>5</sup>C<sup>2</sup></b>
5	<b>D<sup>5</sup>E<sup>3</sup>G<sup>4</sup></b>

Closed					
A	B	C	D	E	G
0					
0	1				
0	1			6	
0	1	2			
...					

Put back to Open

# Open List and Closed List



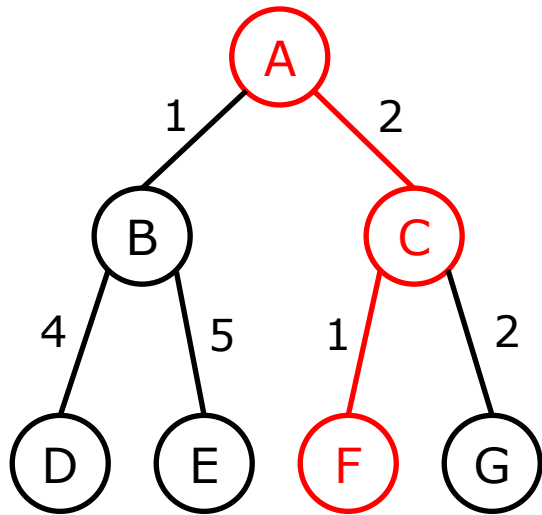
Step	Open
0	
1	<b>A<sup>0</sup></b>
2	<b>B<sup>1</sup>C<sup>2</sup></b>
3	<b>D<sup>5</sup>E<sup>6</sup>C<sup>2</sup></b>
4	<b>D<sup>5</sup>C<sup>2</sup></b>
5	<b>D<sup>5</sup>E<sup>3</sup>G<sup>4</sup></b>

Open + Closed					
A	B	C	D	E	G
∞	∞	∞	∞	∞	∞
0	∞	∞	∞	∞	∞
0	1	2	∞	∞	∞
0	1	2	5	6	∞
0	1	2	5	6	∞
0	1	2	5	3	∞
...					

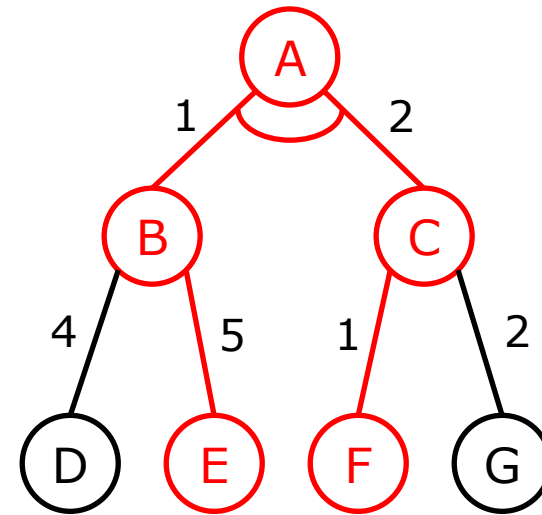
# Search Strategies

- Dijkstra = Best First Graph Search
- A\* = Dijkstra + Heuristics (addition)
- Depth First Branch and Bound (DFBnB) = Depth First + Heuristics + Pruning

# 5. And-Or Search Space

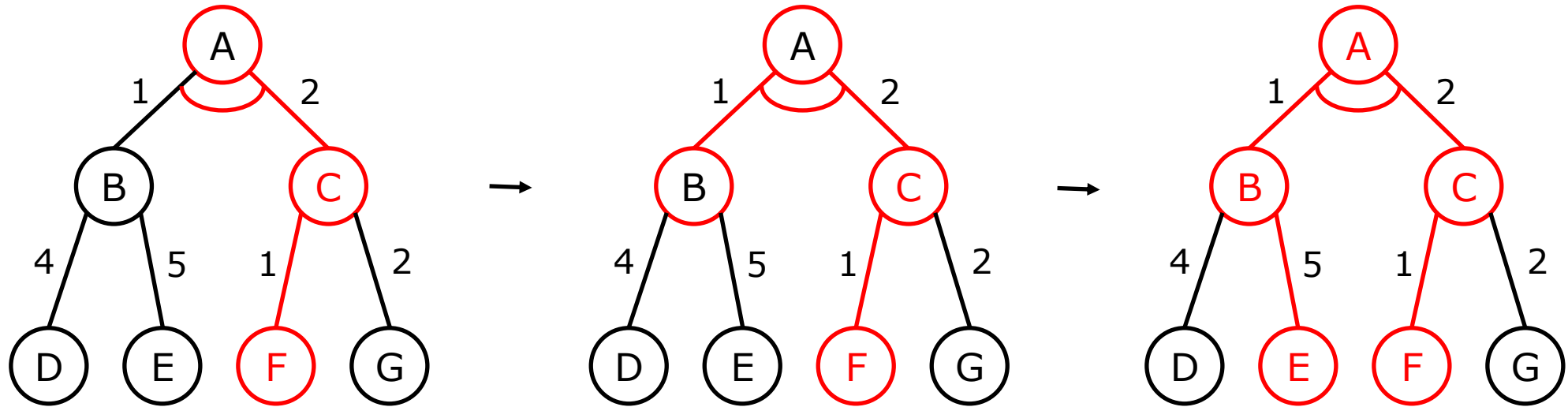


Or Search



And-Or Search

# Labeling Procedure



# Configurational Search Strategies

Deciding a search strategy:

Depth or Best?

One solution or M solutions?

Heuristics? Pruning?

Tree or Graph?

And-or?