CSC 412 Machine Learning and Knowledge Discovery

Logistic Regression

1 $\sigma'(z) =$

$$\boxed{\sigma(z) = \frac{1}{1+e^{-z}}}$$

$$\begin{aligned}
\sigma'(z) &= \frac{\mathrm{d}}{\mathrm{d}z}\Big(\frac{1}{1+e^{-z}}\Big) \\
&= \frac{\mathrm{d}}{\mathrm{d}z}\Big((1+e^{-z})^{-1}\Big) \\
&= (-1)\cdot(1+e^{-z})^{-2}\cdot\frac{\mathrm{d}}{\mathrm{d}z}(1+e^{-z}) \\
&= -\frac{1}{(1+e^{-z})^2}\cdot\frac{\mathrm{d}}{\mathrm{d}z}(1+e^{-z}) \\
&= -\frac{1}{(1+e^{-z})^2}\cdot\big(e^{-z}\big)\cdot\frac{\mathrm{d}}{\mathrm{d}z}(-z) \\
&= -\frac{1}{(1+e^{-z})^2}\cdot\big(e^{-z}\big)\cdot(-1) \\
&= \frac{e^{-z}}{(1+e^{-z})^2} \\
&= \frac{1}{1+e^{-z}}\cdot\frac{e^{-z}}{1+e^{-z}} \\
&= \frac{1}{1+e^{-z}}\cdot\frac{(1+e^{-z})-1}{1+e^{-z}} \\
&= \frac{1}{1+e^{-z}}\cdot\Big(\frac{1+e^{-z}}{1+e^{-z}}-\frac{1}{1+e^{-z}}\Big) \\
&= \frac{1}{1+e^{-z}}\cdot\Big(1-\frac{1}{1+e^{-z}}\Big) \\
&= \sigma(z)\big(1-\sigma(z)\big)
\end{aligned}$$

So,

$$\boxed{\sigma'(z) = \sigma(z)\big(1-\sigma(z)\big)}$$