Fringe-adjusted joint transform correlation

Mohammad S. Alam and Mohammad A. Karim

Improved correlation discrimination is achieved by using a fringe-adjusted joint transform correlator (JTC). This technique is found to yield significantly better correlation output than the classical and binary JTC's for input scenes involving single as well as multiple objects. It also avoids the computation-intensive Fourier-plane joint power spectrum binarization processing of a binary JTC. Two optical implementations for the proposed technique are also suggested.

Introduction

Recent techniques used for optical pattern recognition may be broadly classified into the VanderLugt-type filter-based correlation and the joint transform correlation. A VanderLugt-type correlator requires a priori fabrication of the filter used in the correlation process, thereby prohibiting real-time operation. In addition, the filter must be accurately aligned along the optical axis in the Fourier plane and requires close positioning between the filter and the Fourier transform of the input. On the other hand, a joint transform correlator (JTC) can be operated at video frame rates and does not require the reference image to be known substantially in advance of performing the correlation process.

One of the main problems associated with the classical JTC is the presence of a strong zero-order peak in the output plane that corresponds to the sum of the autocorrelations of the reference and the input signals and almost overshadows the desired correlation peaks. For a single noise-free target, for example, the zero-order peak is at least four times stronger than the cross correlation peaks. This situation becomes more bizarre in the presence of noise in the input scene. In a real implementation, such a zero-order peak may oversaturate the output detector and cause strong spurious reflections. In specific situations where the zero-order term is confined to a narrow region, however, an optical stop can be effectively used at the center of the output plane to overcome this problem.

Recently Javidi and Kuo proposed a binary JTC where the joint power spectrum (JPS) is binarized, based on a hard-clipping nonlinearity in the Fourier plane, to only two values (+1 and −1) before applying the inverse Fourier-transform operation. When compared with the classical JTC, a binary JTC is found to yield superior correlation peak intensity, correlation width, and discrimination sensitivity. The main problem with a binary JTC is the computation time required for the determination of the threshold value used for binarizing the JPS, which acts as a constraint on the system-processing speed. Also the binarization process introduces harmonic correlation peaks, and a portion of the correlation plane energy is distributed among these higher-order harmonic terms. In addition, the higher-order harmonic terms may yield false alarms or may result in misses, thereby complicating the target-detection process.

Most recently a JTC based on an amplitude-modulated filter (AMF) was reported. In this technique, the JPS is multiplied by the AMF before the inverse Fourier transform is applied to yield the correlation output. For single noise-free targets, the AMF-based JTC is found to yield better correlation performance than the classical and binary JTC's. However, the proposed AMF may produce high optical gain for smaller values of the reference signal power spectrum, which may actually degrade the noise performance of the JTC. Johnson et al., reported a JTC that uses Fourier-plane JPS apodization. In this technique, one uses an expensive phase-only spatial light modulator (SLM) at the Fourier plane, and the analysis/simulation results are applicable to only those reference image JPS's that do not contain any zeros. To alleviate the problems and at the same time to increase the autocorrelation peak intensity, accordingly, we propose in this paper a fringe-adjusted JTC in which a real-valued filter called a fringe-adjusted filter (FAF) is used. The performance of the fringe-adjusted JTC is investigated with computer simulation that uses both noise-
free and noisy input images. The proposed scheme has been found to yield better results than the classical and binary JTC’s while avoiding the computation-intensive Fourier-plane processing of a binary JTC.

Analysis

Joint Transform Correlation

The proposed real-time JTC is shown in Fig. 1, where the reference and the input scene are introduced in the input plane by use of an SLM such as a liquid crystal television. Assume that \( r(x, y + y') \) represents the reference image and that \( t(x, y - y') \) represents the input scene in the input plane separated by a distance \( 2y' \) along the \( y \) axis. The input joint image \( f(x, y) \) can be expressed as

\[
f(x, y) = r(x, y + y') + t(x, y - y').
\]

(1)

Lens L1 performs the Fourier transform of \( f(x, y) \), which is given by

\[
F(u, v) = IR(u, v) \exp[\pm kr(u, v)] \exp(jvy') + IT(u, v) \exp[\mp t(u, v)] \exp(-jvy').
\]

(2)

where \( IR(u, v) \) and \( IT(u, v) \) are the amplitudes and \( kr(u, v) \) and \( t(u, v) \) are the phases of the Fourier transforms of \( r(x, y) \) and \( t(x, y) \), respectively. \( u \) and \( v \) are mutually independent frequency domain variables scaled by a factor \( 2\pi/\lambda f \), \( \lambda \) is the wavelength of the collimating light, and \( f \) is the focal length of the Fourier-transforming lenses L1 and L2. The intensity of the complex light distribution produced in the back focal plane of L1, called the JPS, is then detected by a square-law detector like a CCD array or a liquid crystal light valve (LCLV), given by

\[
|F(u, v)|^2 = |R(u, v)|^2 + |T(u, v)|^2 + 2|R(u, v)||T(u, v)| \cos[\phi_r(u, v) - \phi_t(u, v) + 2vy'].
\]

(3)

In a classical JTC, the JPS is Fourier transformed by lens L2 to yield the correlation output. However, in a binary JTC the JPS is first binarized according to a threshold value before taking the inverse Fourier transform of the JPS: \( T_f \)

\[
|F(u, v)|^2 = \begin{cases} 
1 & \text{if } |F(u, v)|^2 \geq T_f \\
0 & \text{otherwise}
\end{cases}
\]

(4)

where \( T_f \) is the JPS binarization threshold, defined by

\[
T_f = \text{median}[|F(u, v)|^2].
\]

(5)

In general, one selects the threshold value by making the histogram of the pixel values of the JPS and then picking the median.\(^6\)

Recently, a JTC based on an amplitude-modulated filter\(^9\) (AMF) has been proposed in which the AMF is defined as

\[
H_{amf}(u, v) = \frac{1}{|R(u, v)|^2}.
\]

(6)

The JPS is multiplied by \( H_{amf}(u, v) \) before the inverse Fourier transform operation is applied to produce the correlation output. This scheme is found to yield better correlation results than a binary JTC. However, the fact that \( |R(u, v)|^{-2} \) may be associated with one or more poles may contribute to other problems. The presence of poles may force the gain of the AMF to approach infinity, thereby imposing a serious restriction on the realization of this technique. Also, smaller values of the AMF may actually accentuate the noise, thereby degrading the noise performance of the system.

Fringe-Adjusted Joint Transform Correlation

To overcome the aforementioned problems of AMF-based JTC, we propose a fringe-adjusted JTC for which the fringe-adjusted filter (FAF) is defined as

\[
H_{faf}(u, v) = \frac{B(u, v)}{A(u, v) + |R(u, v)|^2},
\]

(7)

where \( A(u, v) \) and \( B(u, v) \) are either constants or functions. When \( B(u, v) \) is properly selected, one can avoid having an optical gain greater than unity. With a very small value of \( A(u, v) \), the pole problem is overcome, while at the same time it is possible to achieve a very high autocorrelation peak. The function \( A(u, v) \) may be used to suppress noise or band limit the signal or both. For example, if the noise power spectrum is known, the \( A(u, v) \) factor may be

Fig. 1. Fringe-adjusted JTC implementation—1. BS, beam splitter.
chosen to suppress the noise spectrum at the Fourier plane. Therefore, in a fringe-adjusted JTC, the amplitude matching is used more effectively to produce sharper and larger correlation peak intensity.

Notice that the filters used in VanderLugt-type correlators involve both magnitude and phase, thus complicating the filter fabrication process. On the other hand, the FAF is a real-valued function because it involves only the intensity (i.e., the JPS) and has no phase terms. Therefore a FAF is more suitable for optical implementation. Also, the computations involving the FAF may be completed long before the input scene is introduced in the input plane of the JTC. Thus the inclusion of the filter does not have any significant detrimental effect on the processing speed of the system. However, an additional spatial light modulator is necessary to display the FAF function, as shown in Fig. 1.

The fringe-adjusted JPS is obtained by multiplying the filter function with the JPS. This multiplication is achieved by displaying the JPS and the FAF in two separate SLM’s placed side by side and then illuminating the SLM’s with the same laser, using a beam splitter and mirror combination, as shown in Fig. 1. Thus the fringe-adjusted JPS may be expressed as

\[ G(u, v) = H_{\text{fad}}(u, v)|F(u, v)|^2 \]

\[ = \left[ \frac{B(u, v)}{A(u, v) + |R(u, v)|^2} \right] \left[ |R(u, v)|^2 + |T(u, v)|^2 \right] + 2|R(u, v)||T(u, v)| \times \cos[\phi_{i}(u, v) - \phi_{i}(u, v) + 2\gamma y']\right]. \]  

When \( B(u, v) = 1 \) and \( |R(u, v)|^2 \gg A(u, v) \), the FAF approaches a perfect real-valued inverse filter, and Eq. (7) may be written as

\[ H_{\text{fad}}(u, v) \approx \frac{1}{|R(u, v)|^2} \times |R(u, v)|^2 \times |T(u, v)| \times \cos[\phi_{i}(u, v) - \phi_{i}(u, v) + 2\gamma y'] \]

If the reference is the same as the target, i.e., \( r(x, y + y') = t(x, y - y') \), the fringe-adjusted JPS for the autocorrelation output, when using Eq. (8), is given by

\[ G(u, v) \approx 2\left[ 1 + \cos[\phi_{i}(u, v) - \phi_{i}(u, v) + 2\gamma y'] \right]. \]  

Lenses L2 performs a Fourier transform of the fringe-adjusted JPS, and from relation (10) it is evident that the autocorrelation output will consist of two delta functions located at \( \pm 2\gamma y' \) and a zero-order term. Notice that the zero-order term is also a delta-functionlike output. Therefore it is simpler to block this term by using an optical stop in the correlation plane.

Multiobject Fringe-Adjusted Joint Transform Correlator

If the input scene contains \( n \) objects \( t_i(x, y - y_i) \), \( t_2(x, y - y_2) \), \ldots, \( t_n(x, y - y_n) \), the input joint image may be expressed as

\[ f(x, y) = r(x, y - y') + \sum_{i=1}^{n} t_i(x, y - y_i). \]

The autocorrelation output will consist of two delta functions located at \( \pm 2\gamma y' \) and a zero-order term. Therefore it is simpler to block this term by using an optical stop in the correlation plane.
correlation output. In Eq. (14) the first and third terms correspond to the zero-order terms diffracted on the optical axis at the correlation plane, the second term corresponds to the autocorrelation between the reference and the input scene object \( t_i(x, y - y_i) \), while the last two terms represent the cross correlation between the reference and nontarget objects and between the input scene objects themselves. In general, the autocorrelation peak intensity decreases with the increase in the number of objects in the input scene.\(^{10}\)

An all-optical implementation with one SLM and one LCLV may also be used to implement the proposed fringe-adjusted JTC, as shown in Fig. 2. The LCLV is used to record the JPS. A portion of the input light is passed through an SLM containing the FAF filter. The light beam originating from the SLM is then used as the readout light beam for the LCLV to yield the fringe-adjusted JPS, as shown in Fig. 2. A subsequent Fourier transform by lens L2 produces the correlation output.

**Simulation Results**

To investigate the performance of the proposed fringe-adjusted JTC, we consider (a) an input scene involving a single object and (b) another input scene involving multiple objects. The simulation tests were performed with a two-dimensional fast Fourier transform routine, and the results were plotted with a three-dimensional plotting routine. The correlation experiments were performed next for the classical, the binary, and the proposed fringe-adjusted JTC's. In all the cases, the correlation peak height is normalized with respect to the total energy of the output plane\(^{11}\) over a scale of 0–255 so that it can be easily represented by 8-bit quantizer levels. For the fringe-adjusted JTC, both \( A(u, v) \) and \( B(u, v) \) are taken to be unity to ensure that the gain of the FAF is less than or equal to unity. Notice that smaller values of \( A(u, v) \) and \(|R(u, v)|^2\) may accentuate the noise component of the input signal power spectrum, thereby degrading the correlation performance.

**Input Scene With Single Object**

For the single-object input scene, a 28 x 28 pixel noise-free image of a tank was used as the reference image, as shown in Fig. 3(a), and a 28 x 28 noisy

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**Fig. 2.** Fringe-adjusted JTC implementation-2. BS, beam splitter.

**Fig. 3.** (a) reference image; (b) input scene containing single object.

**Fig. 4.** Correlation output corresponding to the classical JTC.
image of the same tank was used as the target image, as shown in Fig. 3(b). These images were combined and zero padded to form a 512 × 512 pixel input joint image so that there was no circular convolution artifact in the correlation output.

The correlation output for the classical JTC is shown in Fig. 4, which shows that it is difficult to detect the target because the autocorrelation outputs are almost buried in the noise floor. Because the classical JTC yields poor correlation results for noisy images, we limit the comparison of the fringe-adjusted JTC with the binary JTC. In a binary JTC, generally the hard-clipping nonlinearity is used for binarizing the JPS. Accordingly, in the simulation for binary JTC, the JPS median is used as the threshold for binarization. The binarized JPS is shown in Fig. 5(a), and the correlation outputs corresponding to the first-order harmonic of the binarized JPS are shown in Fig. 5(b). When binary JTC is compared with the classical JTC, we observe from Fig. 5(b) that the autocorrelation peak intensity has an increased manifold and that the target can be detected without ambiguity. However, binary JTC requires computation-intensive Fourier-plane processing for the determination of the threshold value and subsequent JPS binarization, which may act as a constraint on system-processing speed.

The FAF used in the fringe-adjusted JTC is shown in Fig. 6(a). The fringe-adjusted JPS, obtained by multiplying the JPS by the FAF, is then Fourier transformed to produce the correlation output, as shown in Fig. 6(b). Comparing Fig. 5(b) with Fig. 6(b), we observe that the autocorrelation peak intensity produced in the fringe-adjusted JTC is approximately 27% higher than that in the binary JTC. The autocorrelation peak-to-sidelobe ratio in the fringe-adjusted JTC is found to be approximately six times higher than that of a binary JTC.

In general, the peak-to-noise ratio is defined as the ratio of the correlation peak intensity to the average
Fig. 7. (a) input joint image containing single reference and multiple objects; (b) correlation output corresponding to Fig. 7(a) for the binary JTC; (c) correlation output corresponding to Fig. 7(a) for the fringe-adjusted JTC.

value of the noise intensity. For the given input joint image, the fringe-adjusted JTC is found to yield a 55% higher peak-to-noise ratio than that of the corresponding binary JTC.

Input Scene With Multiple Objects

Next we investigated the performances of the fringe-adjusted and binary JTC’s when the input scene contained multiple objects. The 512 × 512 pixel input joint image, shown in Fig. 7(a), consists of a number of aircraft, with the reference image separated from the object scene, which contains a group of several different aircraft. Note that one of the input scene aircraft is identical with the reference aircraft. The symbols in this image were chosen because they have complex shapes, assuming that a more complex shape would cross correlate less with dissimilar shapes and correlate more with a similar shape. The correlation outputs corresponding to Fig. 7(a) for the binary and fringe-adjusted JTC’s are shown respectively in Figs. 7(b) and 7(c). Comparing Fig. 7(b) with Fig. 7(c), we observe that the fringe-adjusted JTC yields 35% higher correlation peak intensity and an autocorrelation peak-to-sidelobe ratio 24 times higher than that of the binary JTC. Also, the ratio of the autocorrelation peak intensity to the highest cross-correlation peak intensity is 85% higher in the fringe-adjusted JTC.

Finally the full width of the autocorrelation peak intensity at half of its maximum is found to be 1 x 1 for both binary and fringe-adjusted JTC’s for input scenes involving single and multiple objects. The simulation results for these input scenes are summarized in Tables 1 and 2, respectively. The numerical results obtained in this research are obviously unique to the images shown in Fig. 3 and Fig. 7(a), respectively. However, our efforts with many other images have yielded similar numerical results.

Conclusion

In this paper we have presented a fringe-adjusted filter based JTC for target detection. This technique is found to yield substantially better correlation output than the classical and binary JTC’s for input scenes involving single as well as multiple objects. The FAF is designed such that it avoids the problems associated with an inverse filter, while producing a high autocorrelation peak intensity. It may also be used to attenuate the noise that is present in the input scene provided that the factor \( A(u, v) \) is selected properly. Computer simulation results show that for noisy input images, the fringe-adjusted JTC yields better correlation peak intensity, peak-to-sidelobe ratio, and peak-to-noise ratio than the classical and binary JTCs. For input scenes involving multiple objects where two or more objects are identical, false autocorrelation peaks may be produced at the output plane. This problem may be alleviated by subtracting the input-only JPS from the JPS at the expense of an additional processing step. The input-only JPS may be obtained by displaying only the input scene in the input plane SLM in the absence of reference image and then recording the JPS. By using the proposed technique, we are able to avoid the computa-

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<th>Table 1. JTC Results for Single Object Case</th>
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<td>Type of JTC</td>
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<td>Binary</td>
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<td>Fringe-adjusted</td>
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\(^a\)Autocorrelation peak intensity.  
\(^b\)Autocorrelation peak-to-sidelobe ratio.  
\(^c\)Autocorrelation peak-to-noise ratio.
tion intensive JPS binarization process associated with binary JTC and the false alarms and misses often associated with the multiobject binary JTC because of JPS binarization. The fringe-adjusted JTC, however, requires an extra SLM to display the FAF filter. Inexpensive SLM’s such as liquid crystal televisions may be used for this purpose. Note that the computation associated with the FAF can be completed long before the input scene is actually introduced into the input plane. Therefore the use of this additional filter may not have any detrimental effect on the system-processing speed.

References